

Computational Thinking

Discrete Mathematics

Number Theory

Topic 01 : Computational Thinking

Logic

Lecture 09 : Selection of Python Tasks

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Graphs and
Networks

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Collections

Enumeration

Relations & Functions

Outline

- Different coding styles — using a result variable or multiple print statements
- Different use cases — code block or functions
- Topic specific tasks — Logic, Sets, Collections, Relations, Functions, Enumeration, ...

Outline

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Motivation

These notes* are an attempt to collate all of the Python tasks that we have discussed during the module, as well some comments on general coding patterns and processes.

To remind you of how we integrate Python into the *Discrete Mathematics* module:

- In the tutorials (online quizzes):
 - Multiple choice questions where you are asked to identify the output (or the purpose of) of some code.
- In the practicals:
 - You are asked to either use existing methods (e.g. set intersection, union, etc.) or implement your own using loops, conditional statements and add/append operations.
- In the end of semester examination:
 - You should expect to be asked to read and execute python code, NOT write it.
 - Python interpretation questions will be similar to those given in the online multiple choice quizzes (with the multiple choices options removed) or similar to those you were asked to write in the practicals.

*This is a work in progress, so expect corrections/changes!

Python Cheat Sheet

Base Types

integer, float, boolean, string, bytes

```
int 163 0 -192 0x3F 0b110
      ^   ^   ^   ^   ^
      int 0  hex binary
float 9.32 0.0 -1.7E-6
      ^   ^   ^
      float 0 10^-6
bool False True
str 'some text' or "some text"
bytes b"text\xfe\775"
```

Container Types

ordered containers — repeatable values

```
list [1,5,3] ["a",1,5,5] [5], []
tuple (1,5,3) "a",1,5,5 (5,) ()


Immutable (non-modifiable values)



```
str "153"
```



key containers — no order, unique keys



```
set {"key1", "key2"}
dict {"key1":value1, "key2":value2} dict(a=3,b="v") set()
```


```

Integer Sequences

```
range([start,] end [,step])
start default is 0 (inclusive), end (exclusive), step default is 1.
range(5) → 0,1,2,3,4
range(2,5) → 2,3,4
range(2,12,3) → 2,5,8,11
range(20,5,-5) → 20,15,10
```

Conversions

```
int('153') → 15
int('3f',16) → 63
int(-11.2468) → -1124000000
int(15.56) → 15
round(15.58,1) → 15.6
float('15.56') → 15.56
bool(x)
str(x)
chr(65) → 'A' code ↔ char ord('A') → 65
list('abc') → ['a','b','c']
dict([(3,'three'), (1,'one')]) → {3:'three', 1:'one'}
set({'one','two'}) → {'one','two'}
'random:data:666'.split(':') → ['random','data','666']
'.'.join(['random','data','666']) → 'random:data:666'
[int(x) for x in ['1','29','-3']] → [1, 29, -3]
```

(Specify base in 2nd parameter)
 (Truncate decimal point)
 (Round to 1 decimal place)
 (False for None, zero or empty containers)
 (String representation of x.)
 (Split string using a separator, str → list of str)
 (Join a list of strings, list of str → str)
 (Convert each element in a collection)

Operations on Sets

```

| .union
& .intersection
- .difference
Methods
s.add(key) s.update(s2)
s.clear() s.remove(key)
```

Generic Operations on Containers

```

min(c) max(c) sum(c) sorted(c)
len(c)
all(c) → True if all items in c evaluate to True, else False.
any(c) → True if at least one item in c evaluate to True, else False.
```

(Number of elements in collection c)

Operations on Lists

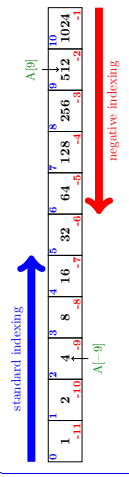
```

a.append(value) a.extend(a2)
a.insert(idx,value) a.pop()
```

Sequence Containers Indexing

```

a[3:6] → [8, 16, 32]
a[1:-1] → [2, 4, 8, 16, 32, 64, 128, 256, 512]
a[::-1] → [1024, 512, 256, 128, 64, 32, 16, 8, 4, 2, 1]
```



Looping over Collections

```

(Loop over values)
for value in A:
    print(value)
break immediately exits loop. continue skips to next iteration.
else block for normal loop exit.
```

(Count and loop over values)

```

for k,value in enumerate(A):
    print(k,value)
break immediately exits loop. continue skips to next iteration.
else block for normal loop exit.
```

While Loop

```

(While loop)
k = 0
while k < len(A):
    print(k, A[k])
    k += 1
```

Initialisation **before** loop.
 update **within** loop.

A copy of this will be attached to your exam paper.

Testing Quantifiers

“x contains the digit 3”

universal quantifier

$$\forall x(x \in A) [x \text{ contains the digit 3}]$$

- To prove **True**, you need to check proposition for every element in A .
- To prove **False**, you just find one element for which the proposition is **False**.

`all(prop(x) for x in A)`

This is a proposition.

It may be **True** or **False**, depending on the choice for x .

E.g., ff $x = 32$ then it is **True**, if $x = 16$ then it is **False**.

existence quantifier

$$\exists x(x \in A) [x \text{ contains the digit 3}]$$

We could implement this as the function:

```
def prop(x):
    return '3' in str(x)
```

- To prove **True**, you just find one element for which the proposition is **True**.
- To prove **False**, you need to check proposition for every element in A .

`any(prop(x) for x in A)`

Different Coding Styles — Single vs Multiple Output Statements

Consider the task of testing if two sets, A and B , are **disjoint**[†]

Algorithm: For each element in A , check that it is in B . If the element is in B then the sets are not disjoint and we stop searching.

Using single output

```

1 A = {2, 3, 5, 7}
2 B = {4, 5, 6}
3
4 result = True
5
6 for x in A:
7     if x in B:
8         result = False
9         break
10
11 print(result)

```

- Single **print** statement.
- So need variable to store desired output.

- Multiple **print** statements.
- No variable needed to store output.
- But need a **loop else** block (only runs if loop ends normally=no break).

Using multiple outputs

```

2 A = {2, 3, 5, 7}
2 B = {4, 5, 6}
3
4 for x in A:
5     if x in B:
6         print(False)
7         break
8 else:
9     print(True)

```

[†]disjoint sets have no elements in common, i.e., their intersection is the empty set.

Different Use Cases — Code Block or Functions (Single Output/Return)

The "Using single output" version when converted to a function has a single **return** statement.

Using single output

```

3
1 A = {2, 3, 5, 7}
2 B = {4, 5, 6}
3
4 result = True
5
6 for x in A:
7     if x in B:
8         result = False
9         break
10
11 print(result)

```

- Single **print** statement.
- So need variable to store desired output.

- Single **return** statement.
- No variable needed to store output.
- Function body is nearly identical to original code block.

Using single return

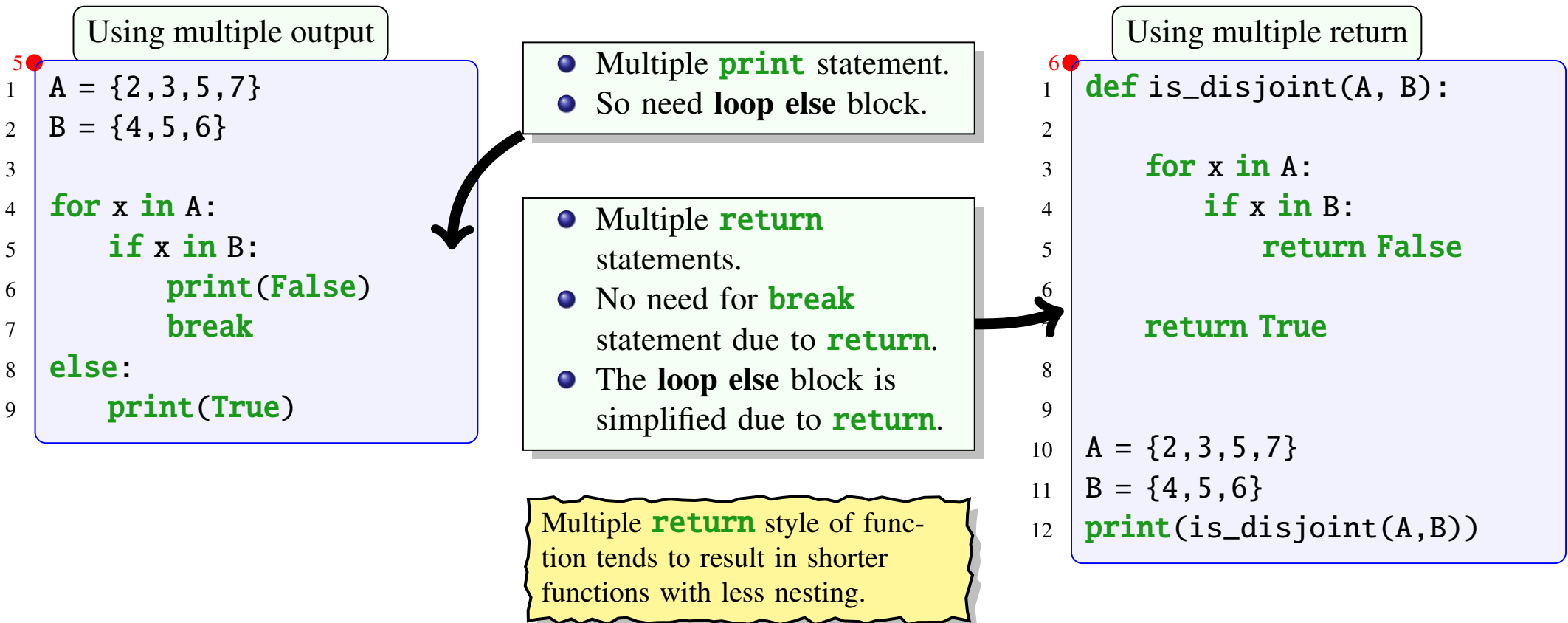
```

4
1 def is_disjoint(A, B):
2
3     result = True
4
5     for x in A:
6         if x in B:
7             result = False
8             break
9
10    return result
11
12
13 A = {2, 3, 5, 7}
14 B = {4, 5, 6}
15 print(is_disjoint(A, B))

```

Different Use Cases — Code Block or Function (Multiple Output/Return)

The "Using multiple outputs" version when converted to a function has a multiple **return** statements.



Selection of Tasks

In the following slides I have collated most of the various tasks that have appeared either in the notes or as questions in the quizzes or the practicals. To keep the number of slides down I only give the "function with multiple return statements" version — this is the coding style that I personally prefer, and use, but be prepared to see the tasks given in the other styles also.

Also, they can often be multiple logically and computationally equivalent solutions to these tasks. So any code presented in the end of semester exams may vary. However, rest assured, I will not intentionally try to obfuscate code, so if you understand the python control statements you should be able to identify the task.

Also, tasks can be combined. For example, combining relation task `is_injective` with function task `is_function` to get code that will check if a given relation is an injective function.

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Set Tasks

Properties

- is equal
- is disjoint
- is subset
- is proper subset

Operations

- intersection
- union
- set difference
- symmetric difference
- Cartesian product

Set Task — `is_equal`

$$A = B$$

TASK

Given two sets, A and B , determine if they are equal*.

ALGORITHM

STEP 1 If size of A is not equal to size of B then sets are not equal.

STEP 2 Else for all elements in A , if element not in B then sets are not equal.

STEP 3 Else sets are equal.

Step 2 is based on the predicate $\forall x (x \in A) [x \in B]$. We can implement this in Python using **any**.

```

7
1 def is_equal(A, B):
2
3     if len(A) != len(B): return False
4
5     for x in A:
6         if x not in B:
7             return False
8
9     return True

```

```

8
1 def is_equal(A, B):
2
3     if len(A) != len(B): return False
4
5     return all(x in B for x in A)

```

*Two sets are equal iff they have the same elements.

Set Task — `is_disjoint`

$$A \cap B = \emptyset$$

TASK

Given two sets, A and B , determine if they are disjoint*.

ALGORITHM

STEP 1 For all elements in A , if element in B then sets are not disjoint.

STEP 2 Else sets are disjoint.

Step 1 is based on the predicate $\forall x (x \in A) [x \notin B]$. We can implement this in Python using **any**.

```

9
1 def is_disjoint(A, B):
2
3     for x in A:
4         if x in B:
5             return False
6
7     return True

```

```

10
1 def is_disjoint(A, B):
2
3     return all(x not in B for x in A)

```

*Two sets are **disjoint** iff they have no elements in common, i.e., intersection is zero.

Set Task — `is_subset`

$$A \subseteq B$$

TASK

Given two sets, A and B , determine if A is a subset* of B .

ALGORITHM

STEP 1 For all elements in A , if element not in B then A is not a subset of B .

STEP 2 Else A is a subset of B .

Step 1 is based on the predicate $\forall x (x \in A) [x \in B]$. We can implement this in Python using **any**.

```

11 ●
1  def is_subset(A, B):
2
3     for x in A:
4         if x not in B:
5             return False
6
7     return True

```

```

12 ●
1  def is_subset(A, B):
2
3     return all(x in B for x in A)

```

* A is a **subset** of B iff every element in A is also in B .

Set Task — `is_proper_subset`

$A \subset B$

TASK

Given two sets, A and B , determine if A is a proper subset* of B .

ALGORITHM

STEP 1 For all elements in A , if element not in B then A is not a subset of B .

STEP 2 Else A is a proper subset of B , iff A is smaller than B .

```

13 ●
1  def is_proper_subset(A, B):
2
3     for x in A:
4         if x not in B:
5             return False
6
7     return len(A) < len(B)

```

```

14 ●
1  def is_proper_subset(A, B):
2
3     return all(x in B for x in A) and len(A) < len(B)

```

* A is a **proper subset** of B iff every element in A is also in B and B contains at least one element that is not in A .

Set Task — intersection

$A \cap B$

TASK

Given two sets, A and B , construct their intersection*, i.e., the set $A \cap B$.

ALGORITHM

STEP 1 Create empty set C .

STEP 2 For all elements in A , if element in B then add element to C .

STEP 3 Return C .

```

15 ●
1  def intersection(A, B):
2
3     C = set()
4
5     for x in A:
6         if x in B:
7             C.add(x)
8
9     return C

```

...or using list comprehension ...

```

16 ●
1  def intersection(A, B):
2
3     return {x for x in A if x in B}

```

* $A \cap B$ is the set of all elements that are in both A and in B .

Set Task — union

 $A \cup B$

TASK

Given two sets, A and B , construct their union*, i.e., the set $A \cup B$.

ALGORITHM

- STEP 1 Create empty set C .
- STEP 2 For all elements in A , add element to C .
- STEP 3 For all elements in B , add element to C .
- STEP 4 Return C .

```

17 ●
1  def union(A, B):
2
3     C = set()
4
5     for x in A:
6         C.add(x)
7
8     for x in B:
9         C.add(x)
10
11    return C

```

We could combine steps 1 and 2 by setting C to be a copy of A using code

```
C = A.copy()
```

Note using $C=A$ does not create a separate set!

* $A \cup B$ is the set of all elements that are in A or in B or in both.

Set Task — set_difference

 $A \setminus B$

TASK

Given two sets, A and B , construct the set difference*, i.e., the set $A \setminus B$.

ALGORITHM

STEP 1 Create empty set C .

STEP 2 For all elements in A , if element not in B then add element to C .

STEP 3 Return C .

```

18 ●
1  def set_difference(A, B):
2
3     C = set()
4
5     for x in A:
6         if x not in B:
7             C.add(x)
8
9     return C

```

```

19 ●
1  def set_difference(A, B):
2
3     return {x for x in A if x not in B}

```

* $A \setminus B$ is the set of all elements that are in A but are not in B .

Set Task — symmetric_difference

$A \oplus B$

TASK

Given two sets, A and B , construct the symmetric difference*, i.e., the set $A \oplus B$.

ALGORITHM

STEP 1 Create empty set C .

STEP 2 For all elements in A , if element not in B then add element to C .

STEP 3 For all elements in B , if element not in A then add element to C .

STEP 4 Return C .

```

20
1  def symmetric_difference(A, B):
2
3     C = set()
4
5     for x in A:
6         if x not in B:
7             C.add(x)
8
9     for x in B:
10        if x not in A:
11            C.add(x)
12
13    return C

```

* $A \oplus B$ is the set of all elements that are in A or in B but not in both sets.

Set Task — cartesian_product

 $A \times B$

TASK

Given two sets, A and B , construct the Cartesian product*, i.e., the set $A \times B$.

ALGORITHM

STEP 1 Create empty set C .

STEP 2 For all elements a in A , for all elements b in B add ordered pair (a, b) to C .

STEP 3 Return C .

```

21 ●
1  def cartesian_product(A, B):
2
3     C = set()
4
5     for a in A:
6         for b in B:
7             C.add( (a,b) )
8
9     return C

```

...or using list comprehension ...

```

22 ●
1  def cartesian_product(A, B):
2
3     C = { (a,b) for a in A for b in B }
4
5     return C

```

* $A \times B$ is the set of all ordered pairs (a, b) where a is an element of A and b is an element of B .

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Relation Tasks

Properties

- is a relation from set A to set B
- is reflexive
- is symmetric
- is transitive
- is irreflexive
- is anti-symmetric
- is asymmetric
- is surjective (onto) (remember if a relation is not onto then it is into)
- is injective (one-to-one)

Relation Task — `is_relation`

TASK

Given relation, R and two sets, A and B , determine if R is a relation* from A to B .

ALGORITHM

STEP 1 For all pairs (a, b) in R , if element a is not in A , then result is **False**.

STEP 2 For all pairs (a, b) in R , if element b is not in B , then result is **False**.

STEP 3 Else R is a relation from A to B .

```

23 ●
1  def is_relation(R, A, B):
2
3      for (a,b) in R:
4          if a not in A:
5              return False
6          if b not in B:
7              return False
8
9      return True

```

The loops in step 1 and step 2 can be merged (as is done in code above).

*A **relation from A to B** is a set of ordered pairs, (a, b) , where a is an element of A and b is an element of B .

Relation Task — is_reflexive

TASK

Given a relation R on a set A , determine if R is reflexive*.

ALGORITHM

STEP 1 For all a in A , if ordered pair (a, a) is not in R , then R is not reflexive.

STEP 2 Else R is reflexive.

Note that we need the set A to test if R is reflexive.

```

24 ●
1  def is_reflexive(R, A):
2
3      for a in A:
4          if (a, a) not in R:
5              return False
6
7      return True

```

An alternative approach is to build a set of all pairs representing the self-loops, and then checking if that set is a subset of R .

Or can use the python **all** predicate.

```

25 ●
1  def is_reflexive(R, A):
2
3      return all( (a, a) in R for a in A )

```

*A relation, R , on A is **reflexive** if all ordered pairs, (a, a) are in R where a is an element of A .

Relation Task — `is_symmetric`

TASK

Given a relation R on a set A , determine if R is symmetric*.

ALGORITHM

STEP 1 For all (a, b) in R , if ordered pair (b, a) is not in R , then R is not symmetric.

STEP 2 Else R is symmetric.

26 ●

```
1 def is_symmetric(R):
2
3     for (a,b) in R:
4         if (b,a) not in R:
5             return False
6
7     return True
```

*A relation, R , on A is **symmetric** if ordered pair (b, a) is in R whenever (a, b) is in R .

Relation Task — is_transitive

TASK

Given a relation R on a set A , determine if R is transitive*.

ALGORITHM

STEP 1 For all two-hop paths in R , if the corresponding one-hop path is not in R , then R is not transitive.

STEP 2 Else R is transitive.

```

27 ●
1  def is_transitive(R):
2
3      for (a,b) in R:
4          for (c,d) in R:
5              if b==c and (a,d) not in R:
6                  return False
7
8      return True
  
```

To find all two-hop paths in R we use a nested for loop. Outer loop finds all pairs (a, b) , inner for loops finds all pairs (c, d) . Whenever $b = c$ we have a two-hop path $(a \rightarrow b = c \rightarrow d)$. Then if one-hop path (a, d) is not in R , R is not transitive.

* A relation, R , on A is **transitive** iff, whenever (a, b) is in R and (b, c) is in R then (a, c) is in R .

Relation Task — is_irreflexive

TASK

Given a relation R on a set A , determine if R is irreflexive*.

ALGORITHM

STEP 1 For all a in A , if ordered pair (a, a) is in R , then R is not irreflexive.

STEP 2 Else R is irreflexive.

```

28 ●
1  def is_irreflexive(R, A):
2
3      for a in A:
4          if (a, a) in R:
5              return False
6
7      return True

```

An alternative approach is to build a set of all pairs representing the self-loops, and then checking if that set and R are disjoint.

Or can use the `pyth` on **any** predicate.

```

29 ●
1  def is_irreflexive(R, A):
2
3      return not any( (a, a) in R for a in A )

```

*A relation, R , on A is **irreflexive** if R does not contain any ordered pairs like (a, a) where a is an element of A .

Relation Task — is_antisymmetric

TASK

Given a relation R on a set A , determine if R is anti-symmetric*.

ALGORITHM

STEP 1 For all pairs $(a, b) \in R$, if (b, a) is also in R and $a \neq b$ then R is not anti-symmetric.

STEP 2 Else R is anti-symmetric.

```

30 ●
1  def is_antisymmetric(R):
2
3     for (a,b) in R:
4         if (b,a) in R and a!=b:
5             return False
6
7     return True

```

* A relation, R , on A is **antisymmetric** iff whenever both (a, b) and (b, a) are in R then $a = b$.

Relation Task — is_asymmetric

TASK

Given a relation R on a set A , determine if R is asymmetric*.

ALGORITHM

STEP 1 For all pairs $(a, b) \in R$, if (b, a) is also in R then R is not asymmetric.

STEP 2 Else R is asymmetric.

```
31 ●
1  def is_asymmetric(R):
2
3      for (a,b) in R:
4          if (b,a) in R:
5              return False
6
7      return True
```

*A relation, R , on A is **asymmetric** iff whenever (a, b) is in R then (b, a) is not in R (including case $a = b$).

Relation Task — is_surjective

TASK

Given a relation R from set A to set B , determine if R is surjective*.

ALGORITHM

STEP 1 Build a set of the second elements in the each of ordered pairs of R .

STEP 2 If this set equals the target (B) then R is surjective, else it is not surjective.

```
32 ●
1  def is_surjective(R,B):
2
3     image = { b for (_,b) in R }
4
5     return image==B
```

*A relation, R , from A to B is **surjective (onto)** iff the image of R equals the target (here target is B). In other words every element in B occurs, at least once, as the second element in the ordered pairs of R .

Relation Task — is_injective

TASK

Given a relation R from set A to set B , determine if R is injective*.

ALGORITHM

STEP 1 Create empty set to store the image of R .

STEP 2 For each ordered pair (a, b) in R if b in image then R is not injective and STOP, else add b to image.

STEP 3 Else R is injective.

*A relation, R , from A to B is **injective (one-to-one)** iff different first elements in the ordered pairs of R implies different second elements in the ordered pairs. In other words every element in B occurs, at most once, as the second element in the ordered pairs of R .

```

33
1  def is_injective(R):
2
3      image = set()
4
5      for (_,b) in R:
6          if b in image:
7              return False
8          image.add(b)
9
10     return True

```

A common trick in python to check if there are repeated items in a collection is to convert to a set and compare size.

```

34
1  def is_injective(R):
2
3      tmp = [b for (_,b) in R]
4
5      return len(tmp)==len(set(tmp))

```

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Function Tasks

Properties

- is given relation from set A to set B a function ?

Function Task — `is_function`

TASK

Given a function R from set A to set B , determine if R is a function*.

ALGORITHM

STEP 1 Create empty set, D , to store the domain of R .

STEP 2 For each ordered pair (a, b) in R if a in D then R is not a function and STOP, else add a to D .

STEP 3 Else R is function iff D equals A .

*A relation, R , from A to B is a **function** iff every element in A appears exactly once as a first element in the ordered pairs of R . In other words every element in A occurs at most once and at least once.

```

35
1  def is_function(R, A):
2
3     D = set()
4     for (a, _) in R:
5         if a in D:
6             return False
7         D.add(a)
8
9     return D==A

```

```

36
1  def is_function(R, A):
2
3     tmp = [ a for (a, _) in R ]
4     D = set(tmp)
5
6     return len(tmp)==len(D) and D==A

```

Notice that step 2 is testing for "at most once", while step 3 is testing for "at least once".